

Table IV. Shifts in Nanometers Relative to *p*-Xylene

<i>n</i>	First transition		Second transition		Third transition	
	Exptl	Calcd	Exptl	Calcd	Exptl	Calcd
10	0	0	+10	+1		-1
9	+3	+1	+11	+2	+4	0
8	+7	+6	+17	+5	+11	+2
7	+16	+18	+24	+16	+13	+5
6	+28	+37	+39	+38	+17	+12
5		+70		+85		+26

observed in other calculations using different models for the alkyl group.²⁰ The experimental values given in Table III for the two higher energy transitions are also somewhat uncertain since they have been corrected to gas phase in order to make a comparison with theoretical values possible. These corrections can, presently, not be accurately calculated and they can therefore be considered as a first approximation. It should also be noted that by including doubly excited configurations, we are treating the ground state and the singly excited states somewhat unequally.²¹ This unbalance can be corrected by including triply excited configurations

(20) B. Roos, *Acta Chem. Scand.*, **21**, 2318 (1967).

(21) T. W. Stuart and N. L. Allinger, *Theor. Chim. Acta*, **10**, 247 (1968).

which can interact with the singly excited configurations. It has been shown²¹ that the inclusion of triply excited configurations in calculations on benzene and toluene, using the VESCF-CI method, only slightly affects the two lowest energy transitions but lowers the energy of the third by about 0.2 eV. Including triply excited configurations in the present calculations should thus give a somewhat better agreement with experiment. (We are presently not prepared to include triply excited configurations in our calculations.)

It should also be remembered that, due to the diminished validity of the π - σ separation approximation for nonplanar compounds, we cannot expect the same accuracy in π -electron calculations on these compounds as in calculations on planar ones.

The experimental red shifts of the second and third transitions as *n* decreases are qualitatively well reproduced by the calculations (Table IV).

Considering the limitations discussed above, the geometries of the [*n*]paracyclophanes obtained by energy minimization account reasonably well for the electronic transitions of these molecules. The predicted spectrum for the still unknown [5]paracyclophane is given in Table III. It shows the expected shifts to the red of all transitions compared to [6]paracyclophane.

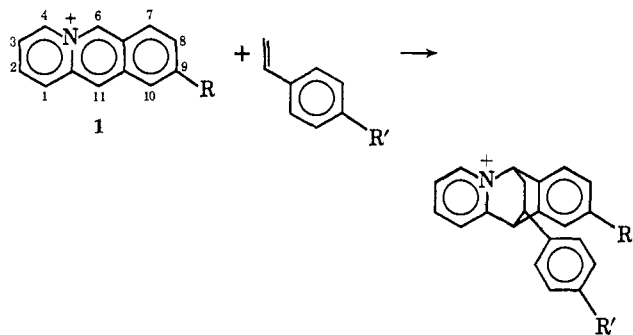
Kinetic Results of a Multiple Substituent Variation in a Polar Cycloaddition. Application of a Frontier Orbital Perturbation Model

N. A. Porter,* I. J. Westerman, T. G. Wallis, and C. K. Bradsher

Contribution from the Paul M. Gross Chemical Laboratories, Duke University, Durham, North Carolina 27706. Received February 4, 1974

Abstract: Rate constants for cycloaddition of 21 different pairs of 9-substituted acridiziniums and para-substituted styrenes are reported. The rate data can be best fitted to a multiple parameter linear free energy equation proposed by Miller ($\log k = \rho_1\sigma_1 + \rho_2\sigma_2 + q\sigma_1\sigma_2 + c$). The polar cycloaddition is discussed in terms of a frontier orbital perturbation model.

The rates of the cycloaddition reaction of substituted acridiziniums (**1**) with para-substituted styrenes have been shown to be strongly influenced by substituents on



either of the reactants.^{1,2} Thus, rate studies of the

(1) I. J. Westerman and C. K. Bradsher, *J. Org. Chem.*, **36**, 969 (1971).

(2) C. K. Bradsher and J. A. Stone, *J. Org. Chem.*, **34**, 1700 (1969).

addition of 9-substituted acridiziniums with para-substituted styrenes² show that electron releasing groups (*e.g.*, $R' = \text{OCH}_3$) on the styrene and electron withdrawing groups (*e.g.*, $R = \text{NO}_2$) on the acridizinium¹ promote the reaction.

The rates of additions of unsubstituted styrene with 9-substituted acridiziniums are best correlated with the σ_p parameter of the 9 substituent¹ (eq 1).

$$\log k = 1.74\sigma_A - 2.35 \quad (1)$$

The rates of cycloadditions of unsubstituted **1** with para-substituted styrenes correlate well with σ^+ for the styrene substituent (eq 2). σ_A is the σ_p constant³ for a

$$\log k = -0.57\sigma_s^+ - 2.24 \quad (2)$$

substituent at position 9 of acridizinium and σ_s^+ is the Brown-Okamoto constant⁴ for the para-styrene substituent.

(3) D. H. McDaniel and H. C. Brown, *J. Org. Chem.*, **23**, 420 (1958).

(4) Y. Okamoto and H. C. Brown, *J. Org. Chem.*, **22**, 485 (1957).

Table I. Rate Constants^a for Cycloaddition of 9-Substituted Acridizinium with Para-Substituted Styrenes at 65°

9 substituent on acridizinium (σ_A)	$k, \times 10^3 \text{ min}^{-1}$					ρ_s
	Para substituent on styrene (σ^+)					
	OCH ₃ (-0.764)	CH ₃ (-0.306)	H (0) ^b	NO ₂ (0.777)	Cl (0.112)	
CH ₃ (-0.17)	9.4	3.7	2.0	1.1		-0.59 ^e
CH(CH ₃) ₂ (-0.151)	9.7	4.4	2.8	1.4		-0.53
H (0) ^c	18.0	8.1	5.0	2.3		-0.57
COOH (0.406)	89.0	33.4	18.1	6.2		-0.74
NO ₂ (0.778)	650	260	105	27		-0.90
Cl (0.227)					9.0	
ρ_A	1.91 ^d	1.87	1.71	1.38		

^a All rate constants have error limits less than $\pm 5\%$. ^b Data in this column previously published, ref 1. ^c Data in this row previously published, ref 2. ^d ρ_A is ρ obtained for addition of para-substituted styrene with fixed substituent with various acridiziniums. For example, $\rho_A = 1.91$ would correspond to ρ 's obtained by reacting paramethoxystyrene with the five substituted acridiziniums. ^e ρ_s is ρ obtained for addition of 9-substituted acridizinium with fixed substituent with various styrenes.

The ρ values found in eq 1 and 2 indicate a transfer of electron density from styrene to acridizinium during the reaction and are consistent with either a stepwise addition of acridizinium to styrene^{2,5} or a concerted "Diels-Alder reaction with inverse electron demand."^{1,2,5-7}

Several reports of substituent effect studies on Diels-Alder reactions have appeared.⁸⁻¹⁰ These studies all involve variation of substituents either on the diene or on the dienophile separately. Charton,^{8a} for example, has examined sets of substituted dienes with various fixed dienophiles as well as sets of fixed dienes with other substituted dienophiles. No study has appeared in which a simultaneous multiple variation of substituents on both diene and dienophile was attempted.

The purpose of this report is to present the results of multiple substituent variation in a cycloaddition reaction where the substituent is allowed to vary simultaneously in both of the components of the cycloaddition.

Experimental Section

All the compounds utilized in the kinetic study had been synthesized previously.^{1,11} The melting points agreed closely with those reported for the authentic samples.

The procedures employed in the rate determinations have been discussed in detail.¹ Briefly, pseudo-first-order conditions were maintained by using a 10:1 ratio of alkene to acridizinium derivative in dimethyl sulfoxide solvent. The rate was followed at a given temperature by the loss of acridizinium's long wavelength absorbance in the visible spectrum. All spectra were measured on a Beckman DB-G uv-visible spectrometer.

The linear correlations and resultant parameters were calculated by means of a nonweighted least-squares program. The statistical treatments and evaluation standards employed were those recommended by Jaffé.¹²

Results

The rate constants for cycloaddition of 21 different pairs of 9-substituted acridiziniums and para-substituted styrenes are presented in Table I.

Analysis of the kinetic data shows a significant Hammett correlation within the rows (constant acridizinium, variable styrene substituent) and columns (constant styrene, variable acridizinium substituent) of the table.

- (5) C. K. Bradsher and J. A. Stone, *J. Org. Chem.*, **33**, 519 (1968).
- (6) (a) J. Sauer, D. Land, and H. Wiest, *Z. Naturforsch. B*, **17**, 206 (1962); (b) J. Sauer and H. Wiest, *Angew. Chem.*, **74**, 353 (1962).
- (7) T. G. Wallis, N. A. Porter, and C. K. Bradsher, *J. Org. Chem.*, **38**, 2917 (1973).
- (8) (a) M. Charton, *J. Org. Chem.*, **31**, 3745 (1966); (b) F. Kaylen and H. Conroy, *ibid.*, **28**, 1593 (1963).
- (9) T. Inukai and T. Kojima, *J. Org. Chem.*, **36**, 924 (1971).
- (10) M. J. S. Dewar and R. S. Pyron, *J. Amer. Chem. Soc.*, **92**, 3098 (1970).
- (11) J. C. Parham, Ph.D. Dissertation, Duke University, 1963.
- (12) H. H. Jaffé, *Chem. Rev.*, **53**, 191 (1953).

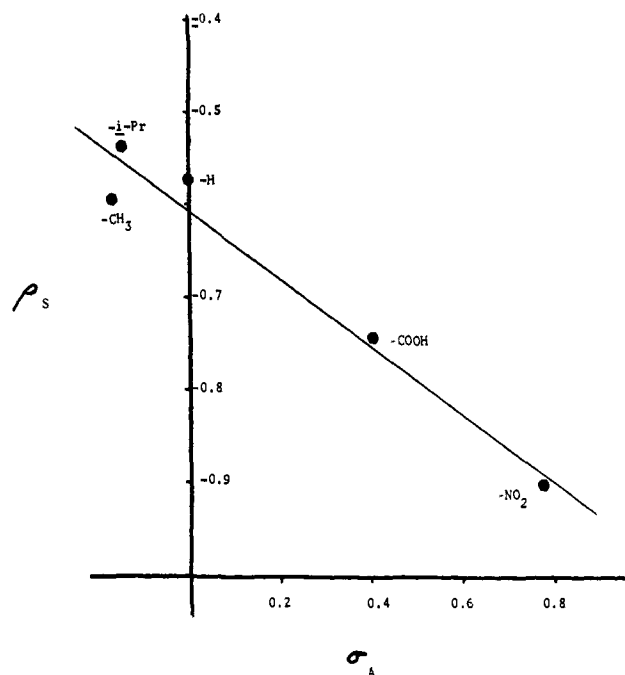


Figure 1. Plot of ρ_s vs. σ_A for cycloaddition of 9-substituted acridiziniums with fixed substituents with four para-substituted styrenes.

An interesting feature was observed in the ρ values obtained for a given acridizinium derivative by varying the styrene component. The absolute value of ρ increased as the electron deficiency of the aromatic cation was intensified. Best fit ρ values for a fixed acridizinium and variable styrene substituent are presented in Table I under the column ρ 's. Examination of the various ρ 's reveals a correlation of ρ 's with σ_A (the value of σ_p for the fixed acridizinium substituent). The best fit line for Figure 1 is described by eq 3.

$$\rho_s = -0.36\sigma_A - 0.606 \quad (3)$$

It should be noted that the best correlation for each of the nine ρ 's calculated above was obtained if σ_p values were used for the substituents on position 9 of the acridizinium and σ^+ values were used for the para substituents on styrene.

Discussion

Two equations have been used to correlate rate data of reactions where more than one Hammett substituent is allowed to vary. The first, an equation of Jaffé's, which was not intended for such a use, has been applied

to a variety of reactions where more than one substituent was allowed to vary (eq 4).

$$\log k = \rho_1\sigma_1 + \rho_2\sigma_2 + \log k_0 \quad (4)$$

Miller later pointed out¹³ that the Jaffé equation is but a special case (where $q = 0$) of a more general equation (eq 5).

$$\log k = \rho_1\sigma_1 + \rho_2\sigma_2 + q\sigma_1\sigma_2 + \log k_0 \quad (5)$$

The Miller equation has been applied in relatively few instances and in many of these examples the improvement in correlation using eq 5 over the fit of eq 4 with experimental data is marginal.¹⁴

When the data of Table I are subjected to a multiple regression analysis¹⁵ to fit the Jaffé equation (eq 5), eq 6 emerges as the best fit.

$$\log k = 1.72\sigma_A - 0.67\sigma_s^+ - 2.27 \quad (6)$$

The fit of (6) to the data is good with R , the multiple regression correlation coefficient,¹⁵ equal to 0.988.

Fitting of the data of Table I to the Miller expression (eq 5) by multiple regression leads to eq 7 as the best

$$\log k = 1.69\sigma_A - 0.61\sigma_s^+ - 0.37\sigma_A\sigma_s^+ - 2.27 \quad (7)$$

fit ($R = 0.993$).

An analysis of the variance on eq 6 (Jaffé equation) was carried out and the results of the analysis are given in Table II.¹⁵ The calculated F ratio of 371 shows a

Table II. Analysis of Variance for Equation 6

Source	Sum of squares	Degrees of freedom	Mean square
Regression on σ_A and σ_s^+	10.767	2	5.384
Residual	0.261	18	0.0145
Total	11.028	20	

significant correlation of the data with σ_A and σ_s^+ at the 99% confidence level.

In Table III is presented the analysis of the variance from eq 7 (the Miller equation).

Table III. Analysis of Variance for Equation 7

Source	Sum of squares	Degrees of freedom	Mean square
Regression of σ_A , σ_s^+ , and $\sigma_A\sigma_s^+$	10.883	3	
Regression ^a due to $\sigma_A\sigma_s^+$	0.116	1	0.116
Residual	0.145	17	0.0085
Total	11.028	20	

^a Reference 15, p 230.

The F ratio for regression due solely to the cross term, $\sigma_A\sigma_s^+$, of 13.5 indicates a significance of $\sigma_A\sigma_s^+$ at the 99% confidence level. That is to say, eq 7 gives a significantly better fit to the data than does eq 6. That the cross term is significant is not surprising since its importance was implied earlier in eq 1-3.

(13) S. I. Miller, *J. Amer. Chem. Soc.*, **81**, 101 (1959).

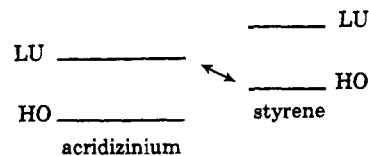
(14) O. Exner, "Advances in Free Energy Relationships," N. B. Chapman and J. Shorter, Ed., Plenum Press, London, 1972.

(15) O. L. Davies, Ed., "Statistical Methods in Research and Production," Hafner Publishing Co., New York, N. Y., 1961, Chapter 8.

Several questions arise from the fact that the Miller expression can be successfully fitted to the rate data for the cycloaddition reactions of substituted acridiziniums and styrenes. (1) Why is a linear free energy relationship (LFER) of any kind found for these cycloaddition reactions? (2) What significance can be attached to the fact that σ^+ for styrene substituents and σ_p for substituents on acridizinium give best fits for these polar cycloadditions? (3) What interpretation can be attached to the fact that inclusion of the $\sigma_A\sigma_s^+$ term significantly improves the expression?

The perturbation HMO method¹⁶ has been applied successfully¹⁷⁻²⁰ to account for regioselectivity and reaction rates in cycloaddition. Scheme I shows the

Scheme I



principal MO interactions between acridizinium and styrene. The interaction between the acridizinium LU orbital and the styrene HO orbital should be dominant and the stabilization energy (SE) of the transition state due to the orbital interaction is given by eq 8 in which

$$SE = \frac{[(C_{ALU}^6 C_{SHO}^\beta + C_{ALU}^{11} C_{SHO}^\alpha)]^2 \beta^2}{[E_{LU}^A - E_{HO}^S]} \quad (8)$$

E_{LU}^A represents the energy of the acridizinium LUMO and E_{HO}^S is the energy of the styrene HOMO. C_{ALU}^6 and C_{ALU}^{11} represent the atomic orbital coefficients of orbitals at positions 6 and 11 in the lowest vacant orbital of acridizinium. C_{SHO}^α and C_{SHO}^β represent atomic orbital coefficients in the highest occupied orbital of styrene at the α and β positions. Sustmann¹⁸ has shown that rates of cycloaddition can be correlated with differences in LU and HO energies thus lending plausibility to the application of eq 8.

If we assume that the energy gap between LU and HO is the major factor in determining SE, as did Sustmann,¹⁸ it follows that the rate of cycloaddition is related to $E_{LU}^A - E_{HO}^S$ by eq 9, where Q is a constant.

$$\log(\text{rate}) = Q/(E_{LU}^A - E_{HO}^S) \quad (9)$$

We next consider the effects of substituents on the HO and LU energy levels. It is well known that the HOMO energy of substituted aromatics can be correlated with the σ^+ parameter of the substituent. Streitwieser²¹ has reviewed the effect of substituents on the ionization potentials of substituted aromatics. Good correlations are found for ionization potentials as a function of σ^+ parameters. More recently, Traylor²² has shown that the energy of charge transfer bands for substituted aromatics and TCNE correlates remarkably well with

(16) K. Fukui, *Fortshr. Chem. Forsch.*, **15**, 1 (1970).

(17) (a) R. Sustmann, *Tetrahedron Lett.*, 2717 (1971); (b) *ibid.*, 2721 (1971).

(18) R. Sustmann and R. Schubert, *Angew. Chem., Int. Ed. Engl.*, **11**, 838 (1972).

(19) (a) K. N. Houk, *J. Amer. Chem. Soc.*, **94**, 8953 (1972); (b) *ibid.*, **95**, 4092 (1973).

(20) N. D. Epiotis, *J. Amer. Chem. Soc.*, **95**, 5625 (1973), and references cited therein.

(21) A. Streitwieser, *Progr. Phys. Org. Chem.*, **1**, 1 (1963).

(22) W. Hanstein, H. J. Bersin, and T. G. Traylor, *J. Amer. Chem. Soc.*, **92**, 829 (1970), and references cited therein.

σ^+ parameters of the substituents. There would thus seem to be good reason to assume a correlation between the HOMO energy level of styrene (the donor orbital) and σ^+ for the substituent.

Furthermore, it would appear that a correlation between the LUMO energy of substituted acridizinium and σ for the substituent is expected. Zuman²³ has tabulated hundreds of examples of half-wave reduction potentials *vs.* σ values for the substituents. σ is found to be a mirror of the LUMO energy of the substituted aromatics.

We thus make the assumption that $E_{LU^A} - E_{HO^S}$ correlates in some way with σ^+ of the styrene substituent and σ for the acridizinium substituent. The crucial question then pertains to the form of the relationship between $E_{LU^A} - E_{HO^S}$ and σ_s^+, σ_A . If we assume that σ_s^+ and σ_A act independently on $E_{LU^A} - E_{HO^S}$, eq 9 can be rewritten as

$$\log(\text{rate}) = \frac{Q}{\Delta E_{H-H} + \delta_A \sigma_A + \delta_s \sigma_s^+} \quad (10)$$

where ΔE_{H-H} is the energy difference between the unsubstituted styrene HO and unsubstituted acridizinium LU and δ_A and δ_s are constants. Expansion of (10) by a Maclaurin series which is truncated at the square term leads ultimately to (11).²⁴

$$\log k = C_0 - \frac{Q}{(\Delta E_{H-H})^2} (\delta_A \sigma_A + \delta_s \sigma_s^+) + \frac{Q}{(\Delta E_{H-H})^3} (\delta_A \sigma_A + \delta_s \sigma_s^+)^2 + \dots$$

$$\log k = C_0 + \rho_A \sigma_A + \rho_s \sigma_s^+ + C_1 (\rho_A \sigma_A + \rho_s \sigma_s^+)^2 + \dots \quad (11)$$

Note that (11) reduces to the Jaffé equation if the C_1 term is dropped. If $\sigma_A \sigma_s^+$ from the C_1 term is included, eq 11 reduces to the Miller expression.

Equation 11 then shows how a LFER for the cycloaddition can be described from the perturbation treatment. (11) is, however, less than satisfactory in two respects. First, there is no justifiable reason for excluding terms in σ_A^2 and $(\sigma_s^+)^2$ from the C_1 term. Inclusion of these higher order terms in a regression analysis of the rate data led to no improvement in the fit. A more fundamental problem with eq 11 is the underlying assumption that the substituent on styrene and the substituent on acridizinium act independently.

In noninteracting styrene and acridizinium (separated molecules) the substituents act on the orbitals in question independently. That is, the substituent on acridizinium will have no effect on E_{HO^S} and likewise the substituent on styrene will not affect E_{LU^A} . On the other hand, as the reaction commences, and the orbital systems interact, the styrene substituent may well begin to affect E_{LU^A} and the acridizinium substituent may also affect E_{HO^S} . The extent of this cross interaction would likely depend upon the extent of any direct substituent interaction already present. In short, intuition suggests that the effects of the substituents on

(23) P. Zuman, "Substituent Effects in Organic Polarography," Plenum Press, New York, N. Y., 1967.

(24) The expansion is valid only if the effect of the substituents on $E_{LU^A} - E_{HO^S}$ is small relative to ΔE_{H-H} .

$E_{LU^A} - E_{HO^S}$ are coupled and equation 10 does not account for this.

Another approach to the problem of the correct form for $E_{LU^A} - E_{HO^S}$ is to work from the best fit experimental eq 7 toward the theoretical perturbation expression (9). In so doing, (12) emerges as a reasonable

$$E_{LU^A} - E_{HO^S} = \Delta E_{H-H} \left(\frac{1}{1 + \delta_A \sigma_A} \right) \left(\frac{1}{1 + \delta_s \sigma_s^+} \right) \quad (12)$$

approximation for $E_{LU^A} - E_{HO^S}$ and the rate expression (eq 13) follows. It is of interest that $1/(1 + \delta_A \sigma_A)$ is

$$\log k = \frac{Q}{\Delta E_{H-H}} (1 + \delta_A \sigma_A + \delta_s \sigma_s^+ + \delta_A \delta_s \sigma_A \sigma_s^+) \quad (13)$$

similar in form to the perturbation expression $C^2 \beta^2 / (E_H - \alpha)$ derived to account for the effect of a single group (such as OCH_3) on the energy of a single orbital with which the group interacts.²⁵ It is also of interest to note that by coupling these effects of the substituents on $E_{LU^A} - E_{HO^S}$ as in (12), the "mutual sensitivities" term $\sigma_A \sigma_s^+$ results.

One final point should be made about the cross term $\sigma_A \sigma_s^+$. We have assumed that the dominant factor in determining the rate of cycloaddition was the energy term in the denominator of eq 8. This assumption has precedent in the work of Sustmann.¹⁸ Any correlation

of the coefficients of the acridizinium LUMO at positions 6 and 11 with σ_p of the 9 substituent and of the styrene HOMO coefficients on the α and β carbons with σ_s^+ would also lead to higher order terms in σ_A and σ_s^+ .

Although we have assumed a concerted cycloaddition, it seems likely that the transition state is a highly unsymmetric species with bond formation at position 6 preceding bond formation at position 11. Berson²⁶ has proposed that a graded continuum may be needed to account for the behavior of systems with varying structure "with a completely symmetrical four-center mechanism at one end and a fully developed two-step mechanism at the other." It seems probable that the reaction of acridizinium with styrene lies toward the stepwise end of the continuum and, if concerted, the transition state is, at least, unsymmetric.

Finally, we should point out that our view of the effect of substituents on the rate of cycloaddition appears to be one which should be generally applicable. Substituents on the donor should generally fit σ^+ best (as is usually found^{4, 27, 28}) and the σ parameter should best fit substituents on the acceptor component.^{27, 29}

(25) M. J. S. Dewar, *J. Amer. Chem. Soc.*, **74**, 3341 (1952).

(26) J. A. Berson and A. Remanick, *J. Amer. Chem. Soc.*, **83**, 4947 (1961).

(27) G. Kresze, J. Firl, H. Zimmer, and U. Wollnick, *Tetrahedron*, **20**, 1605 (1964).

(28) G. Kresze, S. Rau, G. Sabelus, and H. Goetz, *Justus Liebig's Ann. Chem.*, **645**, 57 (1961).

(29) I. Benghiat and E. I. Becker, *J. Org. Chem.*, **23**, 885 (1958).